Exam. Code : 103202
Subject Code : 1049

B.A./B.Sc. 2nd Semester

MATHEMATICS

Paper—II (Calculus)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions each from sections A and B. All questions carry equal marks.

SECTION—A

1. (a) Discuss the continuity of the function

$$f(x, y) = \begin{cases} \dfrac{x^2}{x^2 + y^2 - x}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

at (0, 0).

(b) Use definition to show that :

$$\lim_{(x, y) \to (1, 2)} (x^2 + y^2) = 5$$

5,5

2. (a) If $x^x y^y z^z = c$, show that $\dfrac{\partial^2 z}{\partial x \partial y} = -(x \log e x)^{-1}$,

when x = y = z.

(b) Let $f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$.

Compute $f_{xy}(0, 0)$, $f_{yx}(0, 0)$. 5,5

3. (a) If $x = u + e^{-v} \sin u$ and $y = v + e^{-v} \cos u$, prove that $\frac{\partial u}{\partial y} = \frac{\partial u}{\partial x}$.

(b) If $z = f(x, y)$ is a homogeneous function of x and y of degree n, then show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$.

5,5

4. (a) Expand $e^x \sin y$ in powers of x and y as far as the terms as third degree.

(b) Show that $f(x, y) = \sin x + \cos y$ is differentiable at every point of R^2. 5,5

5. (a) Find the envelope of the family of lines $y = mx + \sqrt{a^2m^2 + b^2}$, m being the parameter.

(b) Find the extreme values of the function :

$$f(x, y) = x^3 + y^3 - 3axy.$$ 5,5

SECTION—B

6. (a) Evaluate $\iint_A x\,y\,dx\,dy$, where A is the region common to the circles $x^2 + y^2 = x$, $x^2 + y^2 = y$.

   (b) Evaluate :

$$\iiint_{x^2+y^2+z^2\le 1} (z^5 + z)\,dx\,dy\,dz$$

5,5

7. (a) Show that $\iint_A \sqrt{x^2+y^2}\,dx\,dy = \frac{38\pi}{3}$, where A is the region in xy-plane bounded by circles $x^2 + y^2 = 4$, $x^2 + y^2 = 9$.

   (b) Evaluate $\iiint xyz(x^2+y^2+z^2)\,dx\,dy\,dz$ over $x^2 + y^2 + z^2 = a^2$ in positive octant. 5,5

8. (a) Find the area enclosed by cardioid $r = a(1 + \cos\theta)$.

   (b) Show that :

$$\iiint_{x^2+y^2+z^2\le 1} (x^2+y^2+z^2)\,dx\,dy\,dz = \frac{4\pi}{5}$$

   over the region $x^2 + y^2 + z^2 \le 1$. 5,5

9. (a) Show that the volume bounded by cylinder $x^2 + y^2 = 4$ and planes $y + z = 4$, $z = 0$ is 16π.

(b) Evaluate by changing the order of integration in

$$\int_0^1 \int_{4y}^4 e^{x^2}\, dx\, dy .$$ 5,5

10. (a) Find the surface area of the sphere :

$$x^2 + y^2 + z^2 = a^2.$$

(b) Show that :

$$\iint\limits_{\substack{y \le x \le 8-y \\ 2 \le y \le 4}} y\, dx\, dy = \frac{32}{3}.$$ 5,5